

## WMAP dark matter constraints on $b$ - $\tau$ Yukawa unification with massive neutrinos

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JHEP04(2009)043

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## WMAP dark matter constraints on $b - \tau$ Yukawa unification with massive neutrinos

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**ABSTRACT:** We revisit the WMAP dark matter constraints on  $b - \tau$  Yukawa Unification in the presence of massive neutrinos. The large lepton mixing indicated by the data may modify the predictions for the bottom quark mass, enabling Yukawa unification also for large  $\tan\beta$ , and for positive  $\mu$  that was previously disfavoured. As a result, the allowed parameter space for neutralino dark matter increases for positive  $\mu$ , particularly for areas with resonant enhancement of the neutralino relic density. On the contrary, a negative  $\mu$  is not easily compatible with large lepton mixing and Dirac neutrino Yukawa couplings, and the WMAP allowed parameter space is in this case strongly constrained.

**KEYWORDS:** Supersymmetry Phenomenology

**ARXIV EPRINT:** [0901.4013](https://arxiv.org/abs/0901.4013)

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**1 Introduction**

Reconciling the Cold Dark Matter (CDM) predictions of supersymmetric models with the stringent constraints from the Wilkinson Microwave Anisotropy Probe (WMAP) has been one of the challenges within the particle physics community in recent years. The amount of CDM deduced from the WMAP data is given by [1]

$$\Omega_{\text{CDM}}h^2 = 0.111^{+0.011}_{-0.015}, \quad (1.1)$$

where  $h$  is the Hubble parameter in units of 100km/s/Mpc and  $\Omega_{\text{CDM}} = \rho_{\text{CDM}}/\rho_c$  the ratio of the matter density of cold dark matter,  $\rho_{\text{CDM}}$ , over the critical density,  $\rho_c$ , that leads to a flat Universe. This puts severe constraints on the type and parameter space of unified theories with Dark Matter candidates. One of the most promising frameworks in this respect is provided by supergravity models that conserve R-parity [2], and thus predict a stable lightest supersymmetric particle (LSP). This particle has to be neutral, and an obvious candidate is the neutralino [3], although gravitinos are also well-motivated [4].<sup>1</sup>

If one imposes Yukawa unification, as expected from Grand Unified Theories (GUTs), the solutions become more predictive and additional constraints are imposed on the model parameters [6–8]. The same holds for bounds from Flavour Changing Neutral Current (FCNC) processes, which have to be included in the analysis and strongly constrain the allowed supersymmetric parameter space [9].

In addition to the above, the neutrino data of the past years provided evidence for the existence of neutrino oscillations and masses, pointing for the first time to physics

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<sup>1</sup>In fact, gravitinos can be dark matter even in theories with R-parity violation, if their lifetime is larger than the age of the universe [5].

beyond the Standard Model [10]. By now, it has been established that the atmospheric and solar mixing angles  $\theta_{23}$  and  $\theta_{12}$  are large and that the squared mass differences are  $\Delta m_{atm}^2 \simeq 2.5 \times 10^{-3} \text{ eV}^2$  and  $\Delta m_{sol}^2 \simeq 8 \times 10^{-5} \text{ eV}^2$  respectively [11]. As expected, however, the additional interactions required to generate neutrino masses also affect the energy dependence of the couplings of the Minimal Supersymmetric Standard Model (MSSM), and thus modify the Yukawa unification predictions. In fact, a first observation had been that the additional interactions of neutrinos to the tau would spoil bottom-tau Unification for small values of  $\tan \beta$  [12], where we are away from any fixed-point structure while the corrections to the bottom quark mass are small. Subsequently, however, it has been realized that the large lepton mixing could naturally restore unification, and even enable Unification for intermediate values of  $\tan \beta$  that were previously disfavoured [13, 14]. It is interesting to note that for the cosmologically favoured area, it is also possible to observe tau flavour violation at the LHC, in the framework of non-minimal supersymmetric Grand Unification [15]. A detailed analysis of model-building aspects, and of the respective effects of SU(5) unification on observables such as the (1-3) neutrino mixing angle, CP-violating/leptogenesis parameters, and Lepton-Flavour-Violating decays and conversions, has been presented for instance in ref. [16]. This sort of analysis, however, is beyond the scope of our current work, where we would like to keep the discussion as model-independent as possible.

In this work, we revisit the issues of Dark Matter and Yukawa Unification taking into account the effects of massive neutrinos, and large lepton mixing, as indicated by the data. As a first step, we extend previous results to large  $\tan \beta$ , finding significant effects on the allowed parameter space and on  $m_b$ . In fact, it turns out that Yukawa Unification in the presence of neutrinos is also compatible with a positive  $\mu$  and large  $\tan \beta$ , unlike what happens if the effects of neutrinos are ignored [17–19]. Passing to the relic density of neutralinos, we studied the consequences of the above, particularly for the  $\chi - \tilde{\tau}$  coannihilation region and for resonances in the  $\chi - \chi$  annihilation channel, finding once more sizeable effects, which are further amplified when large lepton mixing is combined with quantum corrections above the GUT scale.

## 2 Massive neutrinos and gauge and Yukawa unification

The most straightforward extension of the Standard Model (SM) that can accommodate neutrino masses is to include three very heavy right-handed neutrino states and assume that the smallness of masses arises from the see-saw mechanism [20]. In this case, the predictions for  $m_b$  and unification clearly get modified. In particular, radiative corrections from the neutrino Yukawa couplings have to be included for renormalization group runs from  $M_{\text{GUT}}$  to  $M_N$  (scale of the heavy right-handed neutrinos) [21]. Below  $M_N$ , right-handed neutrinos decouple from the spectrum and an effective see-saw mechanism is operative (with the neutrino mass operator running down to low energies). The relevant equations are given in [22] and are summarized in the appendix.

In addition, if the GUT scale lies significantly below a scale  $M_X$ , at which gravitational effects can no longer be neglected, the renormalization of couplings at scales between  $M_X$  and  $M_{\text{GUT}}$  may induce additional effects to the running. The simplest such example is

provided within the framework of the minimal supersymmetric SU(5) GUT, and the Renormalization Group Equations (RGE's) in this case are given in [23] and are also summarized in the appendix. These runs affect Yukawa unification via the supersymmetric corrections to the bottom quark mass; however, since in the leading-logarithmic approximation the induced modifications to soft masses are proportional to the  $V_{\text{CKM}}$  mixing [23], they are significantly suppressed.

Nevertheless, it has been realized that the influence of the runs above the GUT scale on the Dark Matter abundance can be very sizeable [24], due to changes in the relation between  $m_{\tilde{\tau}}$  and  $m_{\chi}$ , which is crucial in the coannihilation area. In dark matter calculations, therefore, the possibility of such effects has to be also analyzed, and compared to the more standard scenarios.

Our starting point will be the SU(5) superpotential at  $M_X$  which (omitting generation indices) is given by

$$\mathcal{W}_X = T^T \lambda_u^\delta T H + T^T \lambda_d \bar{F} \bar{H} + \bar{F}^T \lambda_N^\delta S H + S^T M_N S \quad (2.1)$$

Here,  $T, \bar{F}$  and  $S$  are the **10**,  $\bar{\mathbf{5}}$  and **1** SU(5) superfields, respectively, and  $H$  and  $\bar{H}$  are the **5** and  $\bar{\mathbf{5}}$  Higgs superfields. The couplings  $\lambda_{u,d,N}$  stand for the up-type quark, down-type quarks/charged lepton and Dirac neutrino Yukawa matrices. The symbol  $\delta$  stands for *diagonal* (the up- and down-type quark Yukawa matrices cannot be diagonalized simultaneously). Finally,  $M_N$  is the right-handed neutrino mass matrix. In addition, we work with the soft SUSY-breaking Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & \tilde{T}^\dagger m_{\mathbf{10}}^2 \tilde{T} + \tilde{\bar{F}}^\dagger m_{\bar{\mathbf{5}}}^2 \tilde{\bar{F}} + \tilde{S}^\dagger m_{\mathbf{1}}^2 \tilde{S} \\ & + m_h^2 h^\dagger h + m_{\bar{h}}^2 \bar{h}^\dagger \bar{h} + M_5 \lambda_{5L} \lambda_{5L} + \text{h.c.} \\ & + \{ \tilde{T}^T A_u \tilde{T} h + \tilde{T}^T A_d \tilde{\bar{F}} \bar{h} + \tilde{\bar{F}}^T A_N \tilde{S} h + \text{h.c.} \} \end{aligned} \quad (2.2)$$

where  $m_{\mathbf{10},\bar{\mathbf{5}},\mathbf{1}}$  are the **10**,  $\bar{\mathbf{5}}$  and **1** SU(5) superfields masses, respectively, and  $m_{h,\bar{h}}$  are the **5** and  $\bar{\mathbf{5}}$  Higgs superfields masses.  $A_{u,d,N}$  are the up-type quark, down-type quark/charged lepton and Dirac neutrino trilinear terms. Finally,  $\lambda_{5L}$  is the SU(5) gaugino and  $M_5$  its soft Majorana mass. Furthermore, we will embed neutrino masses in a minimal supergravity scenario (mSUGRA) where universal boundary conditions are assumed at the gravitational scale  $M_X$ :

$$m_{\mathbf{10}}^2 = m_{\bar{\mathbf{5}}}^2 = m_{\mathbf{1}}^2 = m_h = m_{\bar{h}} = m_0^2, \quad (2.3)$$

$$A_f = a_0 \lambda_f, \quad f = \{u, d, N\}. \quad (2.4)$$

When  $M_X \rightarrow M_{\text{GUT}}$ , SUSY being broken at the  $M_X$  scale with universal soft terms, this scenario will converge towards the CMSSM (extended with a see-saw mechanism capable of generating small neutrino masses).

The physical values of the masses will be obtained by integrating the renormalization group equations from  $M_X$  down to low energies. In the first part of the runs we work with the RGE's of SU(5) from the high energy scale  $M_X$  down to  $M_{\text{GUT}}$ . Subsequently, we run the MSSM RGE's, supplemented with right-handed neutrinos, from  $M_{\text{GUT}}$  to the

right-handed neutrino scale,  $M_N$ . At this scale, right-handed neutrinos decouple from the spectrum, leaving us with the MSSM RGE's and the see-saw effective operator (which is also renormalized down to the low energy scale). For the MSSM RGE's we use the two-loop SUSY renormalization group equations [25] except for the trilinear terms, the gaugino and the sfermion masses, which are calculated at the one-loop level.

Electroweak Symmetry breaking (EWSB) occurs at the scale  $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1} \cdot m_{\tilde{t}_2}}$  (where the one-loop contributions to the scalar potential are less relevant). At this scale we incorporate the SUSY threshold corrections to  $m_b$ ,  $m_\tau$  and  $m_t$  by redefining the corresponding Yukawa couplings as done in ref. [17]. From  $M_{\text{SUSY}}$  to  $M_Z$  we use the SM RGE's. The running top mass is calculated at its scale iteratively, removing  $\lambda_t$  along with its derivative from the remaining running from  $m_t$  to  $M_Z$ .

The MSSM parameters obtained with the above procedure are used to evaluate the SUSY spectrum including loop corrections by using *micromegas* [26]. In the CMSSM limit and in the range of parameters considered in the present work, our procedure yields similar results as those obtained with the code *Suspect* [27] for the numerical integration of the RGE's.

For every set of input parameters, we perform an iterative integration of the RGE's (back and forth, from high to low energies) to evaluate the different scales. We define  $M_{\text{GUT}}$  as the meeting point of  $\alpha_1$  and  $\alpha_2$  and use this scale to find the unified value of the couplings,  $\alpha_U$ , for the SU(5) runs above GUT. The value  $\alpha_3(M_{\text{GUT}})$  is obtained from  $\alpha_3(M_Z)$  (its deviation from  $\alpha_U$  being very small). The third generation Yukawa couplings are obtained by using the following set of values:  $m_\tau(M_Z) = 1.7463 \text{ GeV}$  (which takes into account the SM radiative corrections) and the top pole mass  $m_t = 172.6 \text{ GeV}$  [28]. The value of  $\lambda_b$  is obtained either by fixing  $m_\tau(M_Z)$  (when running RGE's from  $M_Z$  up to  $M_X$ ) or through its relation with  $\lambda_\tau$  at the GUT scale (when running from  $M_X$  down to  $M_Z$ ).

Note that we use the standard RGE's in  $\overline{DR}$  scheme, while the bottom mass experimental determination is usually given as the  $\overline{MS}$  running mass,  $m_b^{\overline{MS}}(\mu = m_b)$  [29]. This implies extrapolating the running mass up to the mass of  $Z^0$  in  $\overline{MS}$ , and subsequently passing into  $\overline{DR}$  with a first-order correction,  $1 - \alpha(M_Z)/3 + \mathcal{O}(\alpha^2)$  [30].

The effects of neutrinos on Yukawa unification crucially depend on the magnitude of the dominant neutrino Dirac Yukawa coupling,  $\lambda_N$ , and consequently the magnitude of  $M_N$  (which determines  $\lambda_N$  through the see-saw conditions). Obviously, for low  $M_N$  and  $\lambda_N$ , the effects are less significant.

Large neutrino Yukawa couplings affect unification by increasing the predicted value of  $m_b(M_Z)$ . This can be understood for small  $\tan\beta$  by simple, semi-analytic expressions [13, 14] since only the top and the Dirac-type neutrino Yukawa couplings ( $\lambda_t$  and  $\lambda_N$ ) may be large at the GUT scale. In this case, the RGE's take simple form

$$\begin{aligned}
 16\pi^2 \frac{d}{dt} \lambda_t &= (6\lambda_t^2 + \lambda_N^2 - G_u) \lambda_t, & 16\pi^2 \frac{d}{dt} \lambda_N &= (4\lambda_N^2 + 3\lambda_t^2 - G_N) \lambda_N \\
 16\pi^2 \frac{d}{dt} \lambda_b &= (\lambda_t^2 - G_d) \lambda_b, & 16\pi^2 \frac{d}{dt} \lambda_\tau &= (\lambda_N^2 - G_\ell) \lambda_\tau
 \end{aligned}
 \tag{2.5}$$

Here,  $\lambda_\alpha$ ,  $\alpha = u, d, \ell, N$ , represent the  $3 \times 3$  Yukawa matrices for the up and down quarks, charged lepton and Dirac neutrinos, and  $G_\alpha = \sum_{i=1}^3 c_\alpha^i g_i(t)^2$  are functions of the gauge

couplings with the coefficients  $c_\alpha^i$ 's as in [12]. Denoting by  $\lambda_{b_0}, \lambda_{\tau_0}$  the  $b$  and  $\tau$  couplings at the unification scale, it is found that

$$\lambda_b(t_N) = \rho \xi_t \frac{\gamma_D}{\gamma_E} \lambda_\tau(t_N), \quad \rho = \frac{\lambda_{b_0}}{\lambda_{\tau_0} \xi_N} \quad (2.6)$$

where  $\gamma_\alpha(t)$  and  $\xi_i$  depend only on gauge and Yukawa couplings. Let us then assume successful  $b - \tau$  unification at  $M_{\text{GUT}}$ , with  $\lambda_{\tau_0} = \lambda_{b_0}$ . In the absence of right-handed neutrinos  $\xi_N \equiv 1$ , thus  $\rho = 1$ . In the presence of them, however,  $\lambda_{\tau_0} = \lambda_{b_0}$  at the GUT scale implies that  $\rho \neq 1$  (since  $\xi_N < 1$ ). To restore  $\rho$  to unity, a deviation from  $b - \tau$  unification would seem to be required. However, large lepton mixing is reconciled with Yukawa unification, by making the simple observation that the  $b - \tau$  equality at the GUT scale refers to the (3,3) entries of the charged lepton and down quark mass matrices. It is then possible to assume mass textures, such that, after the diagonalization at the GUT scale, the  $(m_\ell^{\text{diag}})_{33}$  and  $(m_d^{\text{diag}})_{33}$  entries are no-longer equal [13]. The simplest example the one with symmetric mass matrices:

$$\mathcal{M}_d^0 \propto A \begin{pmatrix} y & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathcal{M}_\ell^0 \propto A \begin{pmatrix} x^2 & x \\ x & 1 \end{pmatrix} \quad (2.7)$$

Here,  $A$  may be identified with  $m_b(M_{\text{GUT}})$ . While the textures ensure equality of the (3,3) elements of the down and charged lepton mass matrices, the eigenvalue of the charged lepton mass matrix is not 1, but  $1 + x^2$ , thus implying that  $\lambda_b \neq \lambda_\tau$  after diagonalization. Within this framework, the issue of Yukawa Unification in the presence of large lepton mixing has been analyzed in [13, 14] for small and moderate values of  $\tan \beta$ .

In our current work, we extend these results to all values of  $\tan \beta$  and both signs of  $\mu$ , taking appropriately into account the large supersymmetric corrections to  $m_b$  [31–33], and using up-to-date experimental bounds. Potentially large lepton mixing effects will be taken into account by imposing the GUT condition  $\lambda_\tau = \lambda_b(1 + \delta)$  where  $\delta$  is determined by requiring that  $m_b(M_Z)$  is in the allowed experimental range. The third generation Dirac neutrino Yukawa coupling  $\lambda_N$  is determined through the see-saw mechanism by assuming  $m_{\nu_3} = 0.05 \text{ eV}$  and a heavy Majorana neutrino scale  $M_N = 3 \times 10^{14} \text{ GeV}$  (a value that ensures that  $\lambda_N$  stays within the perturbative regime). On the other hand, the behaviour associated with the absence of a Dirac neutrino coupling should be recovered by appropriately lowering  $M_N$  and thus  $\lambda_N$ . The resulting effects to the allowed parameter space for unification are significant, and give interesting information on the magnitude and origin of lepton mixing. We then proceed to discuss in detail the cosmological implications in the framework of the WMAP data.

### 3 Yukawa unification in SUSY models

While gauge unification is considered as one of the most attractive features of Supersymmetry, the relations among Yukawa couplings derived by embedding  $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$  in a larger gauge group are less clear. The charged fermion mass hierarchies indicate that

any mass correlations induced by a unified gauge symmetry will be more explicitly manifest in the relation between the Yukawa couplings of the third generation,  $\lambda_t$ ,  $\lambda_b$ ,  $\lambda_\tau$ . How this works depends on the theoretical framework that defines the initial conditions. In SO(10), for instance, all fermions are in the same representation, thus, in the simplest realizations, neutrino couplings will be unified with the rest. In SU(5), the field structure is  $(Q, u^c, \ell^c)_i \in \mathbf{10}$  of SU(5) and  $(L, d^c)_i \in \bar{\mathbf{5}}$ , only implying  $b - \tau$  unification; neutrinos are singlets and thus there is a freedom of choice (although the fact that they couple to the same Higgs as the up-type quarks could be a motivation for some link with the top coupling). Clearly, the more restrictive the unification schemes, the stronger the correlations between quarks and leptons.

In supersymmetric models, unification turns out to be very sensitive to the model parameters. Among others, the compatibility of  $b - \tau$  unification with the observed  $b$  and  $\tau$  masses has a sensitive dependence on the sign of the Higgs mixing parameter,  $\mu$ , and on the details of the superparticle spectrum [34]. Moreover, the universality condition on the soft terms can be removed as in [8], resulting in an enhancement of the allowed parameter space. In fact, it turns out that full third generation Yukawa Unification  $\lambda_t = \lambda_b = \lambda_\tau$  in the CMSSM fails to accurately predict the experimental values of the third generation fermion masses, unless one goes to a large  $\tan \beta$  regime with a heavy spectrum and a dark matter abundance that would tend to overclose the universe.

To correctly obtain pole masses within this framework, the standard model and supersymmetric threshold corrections have to be included; for the bottom quark, these corrections result to a  $\Delta m_b$  that can be very large, particularly for large values of  $\tan \beta$ . In mSUGRA-like scenarios, with universal soft terms at  $M_X \geq M_{\text{GUT}}$ , in the absence of phases,  $\Delta m_b$  has the sign of  $\mu$  [31–33], which is required in order to obtain a  $b$ -quark mass in the allowed range in models with  $b - \tau$  unification. However, this also results to a positive supersymmetric contribution to  $\text{BR}(b \rightarrow s\gamma)$ , which can be compatible with the bounds only for a heavy sparticle spectrum.

The allowed parameter space is nevertheless extremely constrained from the bounds on Flavour Changing Neutral Currents, and the new bounds on  $b \rightarrow s\gamma$  [35] are crucial for the whole discussion and comparisons with the SM prediction [36]. The  $g_\mu - 2$  constraints from Brookhaven National Laboratory (BbNL) [37] are also relevant since the supersymmetric contribution to the muon anomalous magnetic moment takes the sign of  $\mu$  in mSUGRA [38]. There are also some uncertainties  $\mathcal{O}(\alpha^2)$  in the SM prediction, due to the hadronic vacuum polarization correction [39]. For a heavy sparticle spectrum the SUSY contribution to  $\Delta a_\mu$  is small with respect to the SM one. However, we do not take into account that constraint derived from  $g_\mu - 2$ , since there is no significant deviation from the SM predictions if the hadronic contribution is calculated using  $\tau$ -decay data (the current experimental value deviates by  $3.4 \sigma$  from the SM prediction if the hadronic vacuum polarization is determined using  $e^+e^-$  annihilation data).

Let us also summarize a few facts on the possible range of the mass of the bottom quark: The  $2\text{-}\sigma$  range for the  $\overline{MS}$  bottom running mass,  $m_b(m_b)$ , is from 4.1-4.4 GeV (with corresponding pole masses from 4.7 to 5 GeV). We also know that  $\alpha_s(M_Z) = 0.1172 \pm 0.002$ , and the central value of  $\alpha_s$  corresponds to  $m_b(M_Z)$  from 2.82 to 3.06 GeV, while the value



$m_b(m_b) = 4.25 \text{ GeV}$  is mapped to  $m_b(M_Z) = 2.92 \text{ GeV}$ . The allowed strip for  $m_b(M_Z)$  moves to  $2.74 \text{ GeV} < m_b(M_Z) < 3.014 \text{ GeV}$  for  $\alpha_s^{\max}$  and to  $2.862 \text{ GeV} < m_b(M_Z) < 3.114 \text{ GeV}$  for  $\alpha_s^{\min}$ .

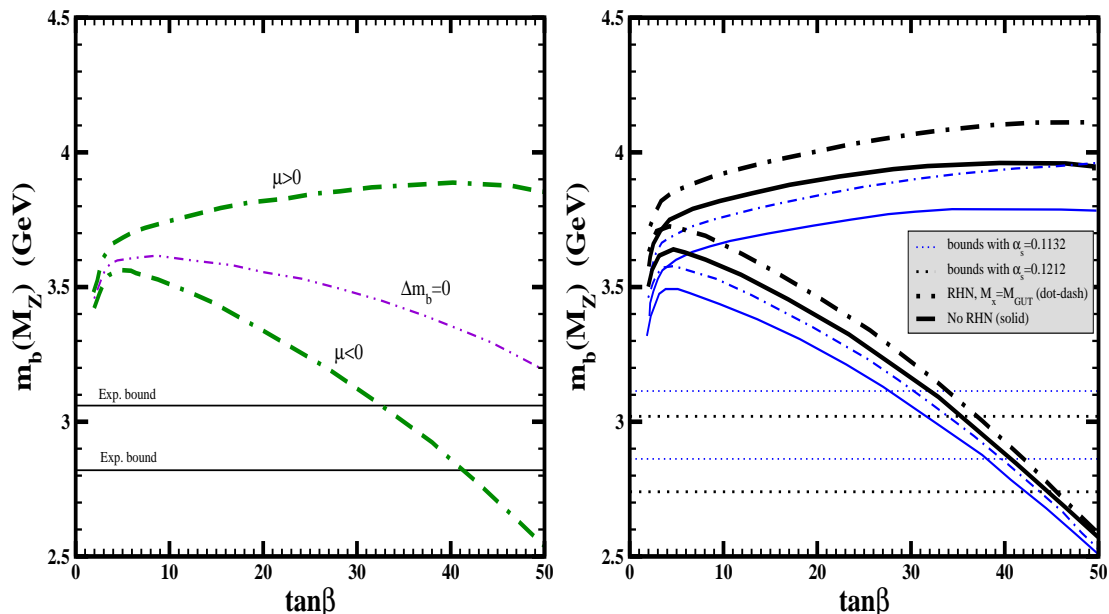
In the left panel of figure 1, we summarize the predictions for  $m_b$  in the CMSSM, where all CP phases are either zero or  $\pi$  (defining the sign of  $\mu$ ). In order to discuss the dependence of  $m_b(M_Z)$  on  $\tan\beta$ , we consider the following set of soft parameters:  $M_{1/2} = 800 \text{ GeV}$ ,  $A_0 = 0$ ,  $m_0 = 600 \text{ GeV}$ . We study this set for both  $\mu > 0$  and  $\mu < 0$ , setting  $\alpha_s(M_z) = 0.1172$ . We also include a reference line without the SUSY corrections to the bottom mass (double-dot-dash line, for  $\Delta m_b = 0$ ). The figure exhibits the known fact that in the absence of phases or large trilinear terms,  $\Delta m_b$  is positive for  $\mu$  positive, and therefore the theoretical prediction for the  $b$  quark pole mass is too high to be reconciled with  $b - \tau$  unification. On the other hand, for  $\mu < 0$ ,  $\Delta m_b$  is negative and the theoretical prediction for the  $b$ -quark mass can lie within the experimental range for values of  $\tan\beta$  between roughly 30 and 40; clearly, for a large  $\tan\beta$  it is mandatory to take into account the large supersymmetric corrections to  $m_b$  [31–33]. The figure also illustrates that, for  $\mu > 0$  and after taken properly into account supersymmetric corrections,  $m_b$  scales very slowly with  $\tan\beta$ . In other words, the renormalization flow for the bottom mass is attracted by some fixed value regardless of the initial conditions [32, 40] (which as we see in the figure turns out to be too large). This *quasi-fixed* point, however, is not generated purely by the renormalization flow given by the RGE, and only appears after including supersymmetric corrections (the curve for  $\Delta m_b = 0$  is not flat in figure 1). Furthermore, for  $\mu < 0$ , there is no fixed point at all.

In the right panel of figure 1 we repeat the analysis in the presence of massive neutrinos, keeping only the third generation couplings (and ignoring lepton mixing effects) from the  $M_{\text{GUT}}$  to the scale of the right-handed neutrino masses,  $M_N$ , and evolve the light neutrino mass operator from this scale down to  $M_Z$ . A large value of the Dirac-type neutrino Yukawa coupling,  $\lambda_N$ , at the GUT scale may arise naturally within the framework of Grand Unification, and its value is determined, through the see-saw mechanism, by demanding a third generation low energy neutrino mass of  $m_{\nu_3} = 0.05 \text{ eV}$  and evaluating the respective heavy neutrino Majorana mass from the see-saw conditions. The predictions for  $m_b(M_Z)$  using the lower and upper bounds of the  $2\text{-}\sigma$  experimental range of  $\alpha_s$  and the corresponding range for  $m_b(M_Z)$  after the evolution of the bounds on  $m_b(m_b)$  are shown for  $M_N = 3 \times 10^{14} \text{ GeV}$ .

We observe that for  $\mu > 0$  the prediction for  $m_b(M_Z)$  is always very large, despite its dependence on the soft terms through  $\Delta m_b$ . For  $\mu < 0$ , there is a window of values of  $\tan\beta$  compatible with asymptotic  $b - \tau$  Yukawa unification. For the values of the soft terms considered in figure 1, the allowed range of  $\tan\beta$  moves from  $27 - 44$  to  $30 - 45$  when we introduce the effect of see-saw neutrinos. Let us stress that  $b - \tau$  Yukawa unification is (is not) compatible with  $m_b$ , for  $\mu < 0$  ( $> 0$ ), regardless on whether or not massive neutrinos are included, when lepton mixing effects are ignored.

#### 4 Yukawa unification and $m_b$ for large lepton mixing

The results are significantly modified once we consider the effects of lepton mixing in the diagonalization and running of couplings from high to low energies. In order to show this,



**Figure 1.** The value of  $m_b(M_Z)$  versus  $\tan\beta$  assuming  $\lambda_b = \lambda_\tau$  at the high scale in the absence (left plot) and presence (right plot) of massive neutrinos and for both signs of  $\mu$  for the following set of parameters:  $M_{1/2} = 800$  GeV,  $A_0 = 0$  GeV,  $m_0 = 600$  GeV. The experimental range of  $m_b$  (horizontal lines) shown in the left panel is computed for the central value of  $\alpha_s(M_Z)$ . In the right panel, the solid lines are obtained within the MSSM, while the dot-dash lines include Dirac Yukawa couplings up to the scale  $M_N = 3 \times 10^{14}$  GeV. Here we take the lowest (blue thin) and highest (black thick) experimental values of  $\alpha_s(M_Z)$ .

we focus on  $b - \tau$  unification within the framework of SU(5) gauge unification and flavour symmetries that provide consistent patterns for mass and mixing hierarchies, and naturally reconcile a small  $V_{CKM}$  mixing with a large charged lepton one. Taking into account the particle content of SU(5) representations (with symmetric up-type mass matrices, and down-type mass matrices that are transpose to the ones for charged leptons), one finds that

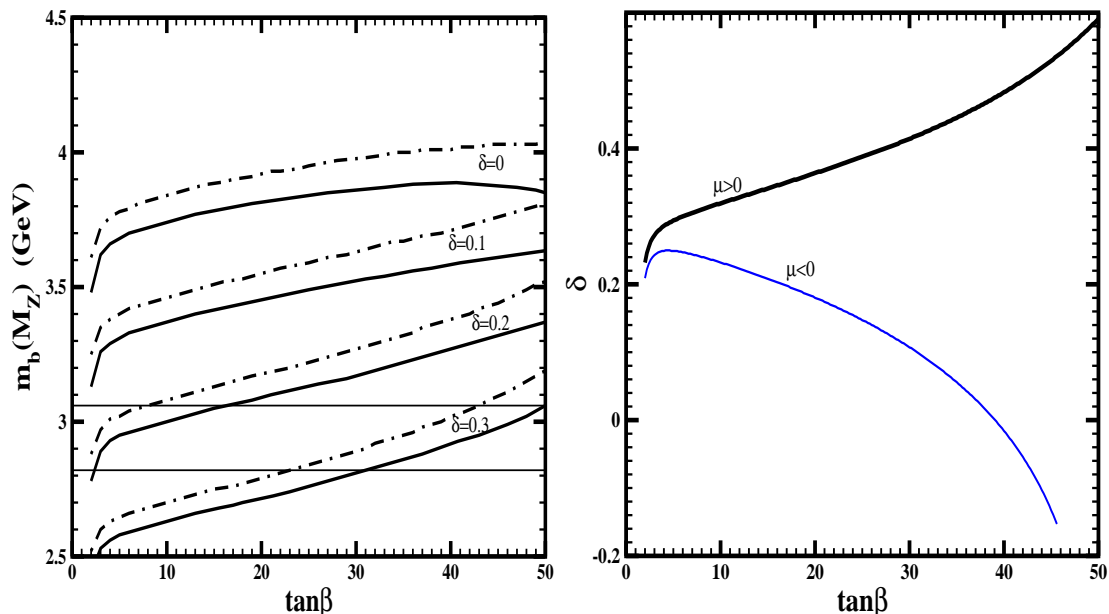
$$\mathcal{M}_u \propto \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^2 & 1 \end{pmatrix}, \quad \mathcal{M}_d^0 \propto A \begin{pmatrix} 0 & 0 \\ x & 1 \end{pmatrix}, \quad \mathcal{M}_\ell^0 \propto A \begin{pmatrix} 0 & x \\ 0 & 1 \end{pmatrix} \quad (4.1)$$

which, after diagonalization, lead to the relation

$$\frac{m_b^0}{1+x^2} = \frac{m_\tau^0}{1-x^2} \rightarrow m_b^0 = m_\tau^0 \left( 1 - \underbrace{2x^2}_\delta + \mathcal{O}(\delta^2) \right) \quad (4.2)$$

where  $\delta$  parameterizes the flavour mixing in the (2,3) charged lepton sector (any additional mixing required to match the data would then arise from the neutrino sector).

In the left panel of figure 2 we show the change of  $m_b$  as a function of  $\tan\beta$ , when the effects from large lepton mixing are correctly considered. Comparing with the previous plots, we see how solutions with positive  $\mu$  are now viable, for the whole range of  $\tan\beta$ . The appropriate size of the parameter  $\delta$  in each case can be determined by imposing the



**Figure 2.** In the left panel, we show  $m_b$  as a function of  $\tan\beta$  for  $\mu > 0$ , without (solid lines) and with (dot dashed) massive neutrinos, in the latter case assuming  $m_{\nu_3} = 0.05\text{eV}$  and  $M_N = 3 \times 10^{14}\text{GeV}$ . In the right panel, we show the required values of  $\delta$  compatible with  $b - \tau$  unification at the central experimental value of  $m_b$  for both signs of  $\mu$ .

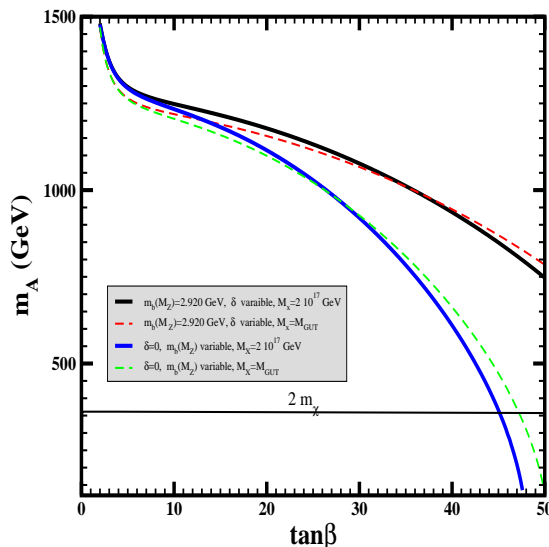
relation  $\lambda_\tau = \lambda_b(1 + \delta)$  at  $M_{\text{GUT}}$  and investigating the values that are required in order to obtain a correct prediction for  $m_b(M_Z)$ . This is shown in the right panel of figure 2, where we demand a value of  $m_b(M_Z)$  at the center of its experimental range, for the central value of  $\alpha_s$ . We checked (although not shown) that the effect from runs above  $M_{\text{GUT}}$  is very small, as expected. We then observe the following:

- Solutions with a positive  $\mu$  are now enabled, for the whole range of  $\tan\beta$ .
- Negative  $\mu$  is also allowed in the whole range of  $\tan\beta$  and requires a smaller mixing parameter  $\delta$  than the  $\mu > 0$  case.

## 5 Yukawa unification and dark matter constraints

In the CMSSM, for choices of the soft terms below the TeV scale, the LSP is mainly Bino-like and the prediction for  $\Omega_\chi h^2$  is typically too large for models that satisfy the experimental constraints on SUSY. These constraints exclude models with relatively small values of  $m_0$  and  $M_{1/2}$  in which neutralino annihilates mainly through sfermion exchange. The values of WMAP can be obtained mainly in two regions:

- $\chi - \tilde{\tau}$  coannihilation region, which occurs for  $m_\chi \sim m_{\tilde{\tau}}$ .
- Resonances in the  $\chi - \chi$  annihilation channel, which occur for  $m_A \sim 2m_\chi$ .



**Figure 3.** Values of the pseudoscalar higgs mass  $m_A$  versus  $\tan\beta$  for the same setting of parameters as in figure 1. In the upper lines the parameter  $\delta$  varies so that  $m_b(M_Z) = 2.921$  GeV, while in the lower lines  $\delta = 0$ . The solid lines correspond to  $M_X = 2 \cdot 10^{17}$  GeV and the dashed lines to  $M_X = M_{\text{GUT}}$ .

Since the above areas are tuned [41], they will inevitably be sensitive to the changes induced by GUT unification and sizeable mixing in the charged lepton sector. In this respect, it is illustrative to first investigate the impact of the parameter  $\delta$  on the value of  $m_A$ . We show this in figure 3, where we present the variation of  $m_A$  with  $\tan\beta$  for  $m_0 = 600$  GeV,  $M_{1/2} = 800$  GeV and  $A_0 = 0$ . In the upper lines the value of  $\delta (\neq 0)$  is fixed by demanding  $m_b(M_Z) = 2.921$  GeV while in the lower ones  $\delta = 0$  with no restrictions on the  $m_b(M_Z)$  prediction. We observe that the upper lines do not meet the resonance condition, while for the lower lines this condition is achieved for values of  $\tan\beta$  in the range 40 – 50. We also see that, assuming soft term universality at a scale  $M_X > M_{\text{GUT}}$ , the runs beyond  $M_{\text{GUT}}$  change only moderately the prediction for  $m_A$ .

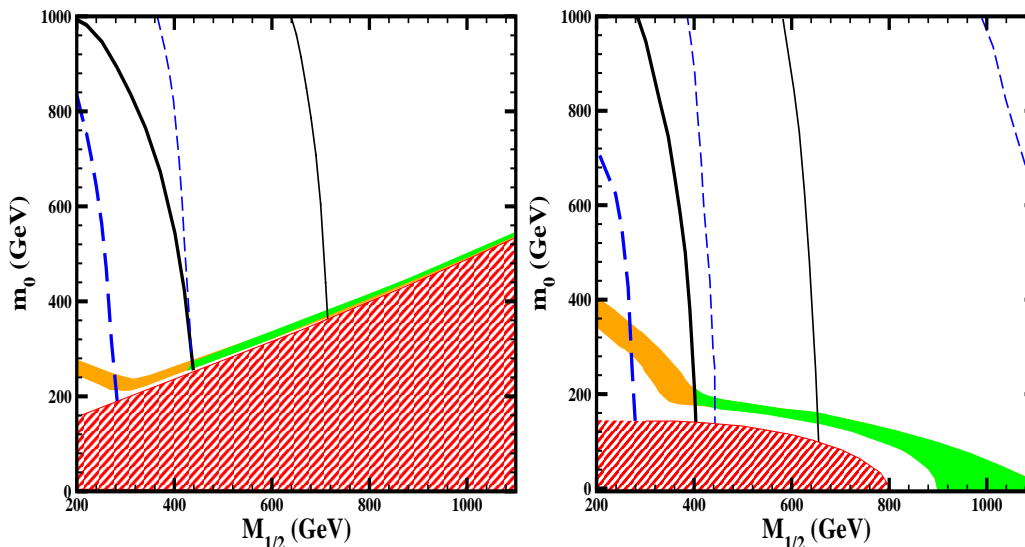
Our next step is a global study of the supersymmetric parameter space by assuming universal soft terms  $m_0$ ,  $M_{1/2}$  and  $A_0$  at some scale  $M_X \geq M_{\text{GUT}}$ , and obtain the mass spectrum by integrating the appropriate RGE's at each energy range, as described above. The physical observables are computed using the code provided by *micromegas* [26]. The  $2\text{-}\sigma$  range for  $\text{BR}(b \rightarrow s\gamma)$  is constructed including an intrinsic  $0.15 \cdot 10^{-4}$  MSSM correction as in ref. [42], leading to

$$2.15 \cdot 10^{-4} < \text{BR}(b \rightarrow s\gamma) < 4.9 \cdot 10^{-4} \tag{5.1}$$

Without including the MSSM intrinsic error, the above range becomes

$$2.8 \cdot 10^{-4} < \text{BR}(b \rightarrow s\gamma) < 4.4 \cdot 10^{-4} \tag{5.2}$$

We take a conservative bound for the mass of the lightest  $CP$ -even Higgs of  $m_h = 114$  GeV and also consider an uncertainty of  $\sim 3$  GeV [43] to its theoretical computation.



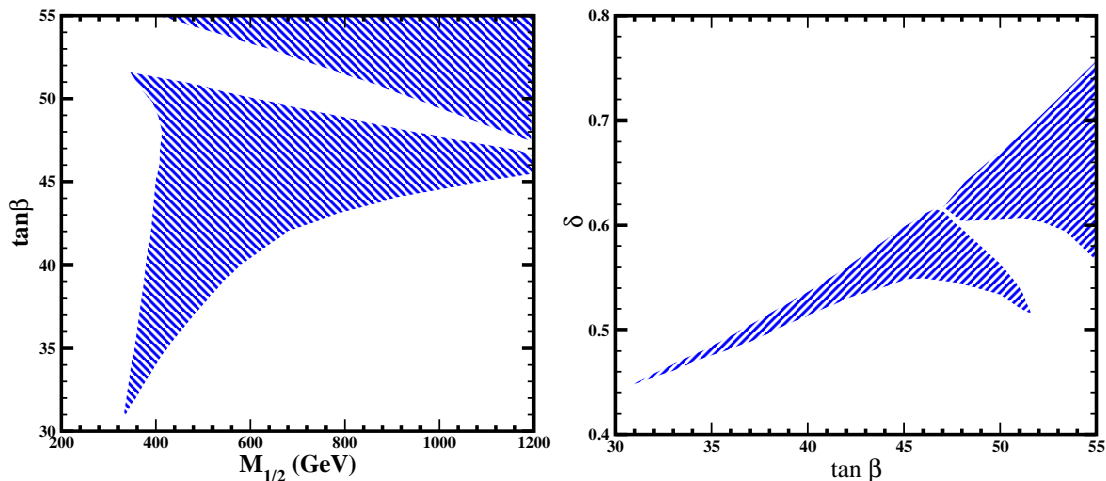
**Figure 4.** WMAP allowed area (green-shaded) for the case of  $\tan\beta = 45$ ,  $\mu > 0$ ,  $A_0 = 0$ , when  $\alpha_s(M_Z)$  and  $m_b(M_Z)$  are set at their central values. The solid black lines indicate the  $b \rightarrow s\gamma$  constraints [thick  $\text{BR}(b \rightarrow s\gamma) = 2.15 \cdot 10^{-4}$ , thin  $\text{BR}(b \rightarrow s\gamma) = 2.8 \cdot 10^{-4}$ ], and the dash blue line the Higgs mass bound [thin  $m_h = 114$  GeV, thick  $m_h = 111$  GeV]. On the left graph,  $M_X = M_{\text{GUT}}$  while on the right panel,  $M_X = 2 \cdot 10^{17}$  GeV.

In figure 4 we present the WMAP favoured area for  $\tan\beta = 45$ , including lepton mixing effects and considering the most tolerant bounds on  $\text{BR}(b \rightarrow s\gamma)$  and  $m_h$  (stricter bounds on these constraints are also displayed). In this example, we keep  $m_b(M_Z) = 2.920$  GeV which corresponds to the evolution of the experimental value of  $m_b$  obtained for the central value  $\alpha_s(M_Z) = 0.1172$ . When  $M_X = M_{\text{GUT}}$  we find the WMAP favored region to be on the familiar CMSSM  $\chi - \tau$  coannihilation area. However, as in [24], we find that the runs corresponding to  $M_X > M_{\text{GUT}}$  have a big impact on the neutralino relic density and figure 4 shows clearly this effect. The large values of the gauge unified coupling  $\alpha_{\text{SU}(5)}$  tend to increase the values of  $m_{\tilde{\tau}}$  in a way that, even if we start with  $m_0 = 0$  at  $M_X$  the model predicts  $m_{\tilde{\tau}} > m_{\tilde{\chi}}$ . This implies that the allowed parameter space with a neutralino LSP is significantly enhanced (green area), while the coannihilation condition becomes harder to achieve; this effect is more visible for large  $\tan\beta$  and is understood since the lightest stau has a mass  $m_{\tilde{\tau}_1} \simeq m_{\tilde{\tau}_{\text{RR}}} + m_{\tilde{\tau}_{\text{LR}}}$ , where

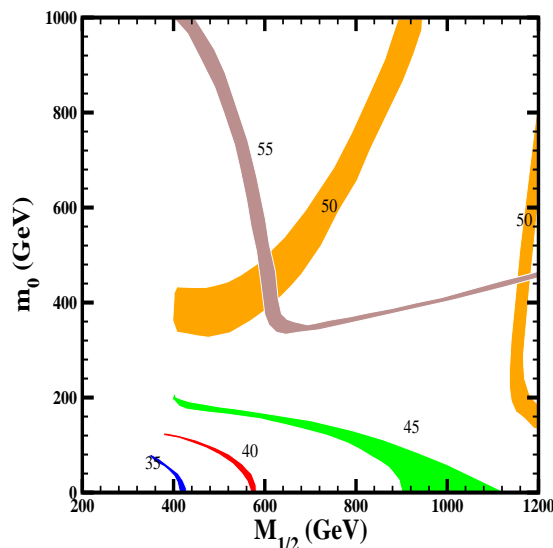
$$m_{\tilde{\tau}_{\text{RR}}}^2 \simeq (1 - \rho_\beta) m_0^2 + 0.3M_{1/2}^2, \quad m_{\tilde{\tau}_{\text{LR}}}^2 \simeq -m_\tau \mu \tan\beta \quad (5.3)$$

$\rho_\beta$  being a positive coefficient dependent on  $\tan\beta$  (in our case  $\rho_\beta < 1$ ) [24]. The increase of  $m_{\tilde{\tau}_1}$  is due to the GUT runnings ( $\sim M_{1/2}^2$ ). The picture for  $\tan\beta = 35$  and  $A_0 = m_0$  has been discussed in [15], where it was shown that the WMAP allowed region can be compatible with observable flavour violation at the LHC.

We stress again that  $b - \tau$  Yukawa unification in the context of the CMSSM requires  $\delta > 0$ . An exhaustive study of the parameter space for the case of  $A_0 = 0$  and keeping the prediction of  $m_b(M_Z) = 2.92$  GeV is presented in figures 5 and 6. The left panel of figure 5



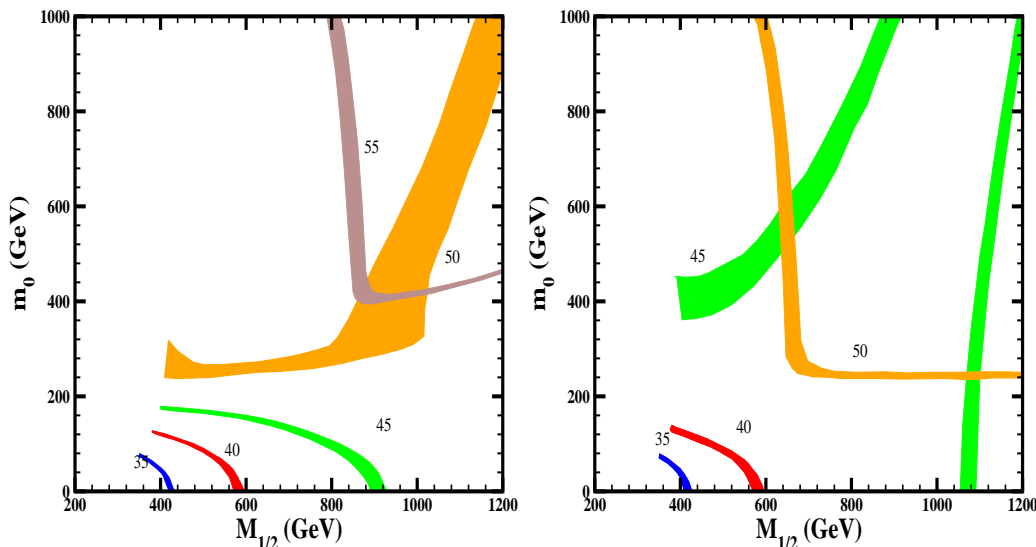
**Figure 5.** The left panel provides a plot equivalent to figure 4 of [24] for the purpose of comparisons; the upper part corresponds to areas with resonant Higgs channels. In the right panel, we show the values of  $\delta$  corresponding to the previous plot; these inevitably implies  $\delta \neq 0$ .



**Figure 6.** WMAP allowed areas for several values of  $\tan\beta$ . We consider  $m_b(M_Z) = 2.92$  GeV,  $A_0 = 0$  GeV and  $\mu > 0$ .

shows that the phenomenological and cosmological constraints are fulfilled for  $\tan\beta \geq 31$  and  $M_{1/2} \geq 330$  GeV respectively. On the other hand, the right panel of figure 5 indicates that sizeable values of the mixing parameter  $\delta$  are needed in order to maintain  $m_b(M_Z)$  in the experimental range and that the required mixing increases with  $\tan\beta$ . Figure 6 shows the  $(m_0, M_{1/2})$  plane for different values of  $\tan\beta$ , taking all constraints into account; it can be seen that the resonant effects start becoming important at  $\tan\beta \geq 45$ .

The impact of varying  $m_b$  and  $\alpha_s$  within their experimental range can be seen in figure 7 which indicates that, for  $\tan\beta$  below 40 there are no significant changes with the variation of  $m_b$  on the WMAP area that lies on the coannihilation region. For larger values



**Figure 7.** Same plot as figure 6, but taking the extreme values of both  $\alpha_s(M_Z)$  and  $m_b(M_Z)$ . On the left panel,  $\alpha_s(M_Z) = 0.121$  and  $m_b(M_Z) = 2.74$  GeV while on the right panel  $\alpha_s(M_Z) = 0.113$  and  $m_b(M_Z) = 3.114$  GeV. In both cases  $M_X = 2 \cdot 10^{17}$  GeV.

of  $\tan \beta$ , however, small changes of the bottom Yukawa coupling due to the modification of  $m_b$  have a significant impact on resonant annihilation, as is manifest by the locations of the bands for  $\tan \beta = 45$  and 50 in the two lower plots.

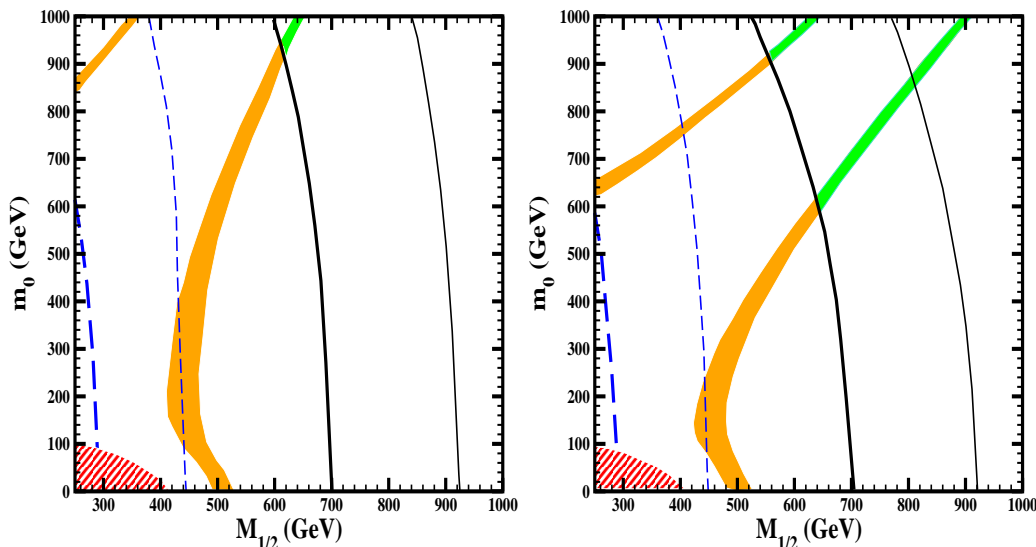
The case  $\mu < 0$  is not yet ruled out by the  $(g - 2)_\mu$  data, since as we mentioned before by using  $\tau$  data, a small negative discrepancy of the experimental measurement as compared to the SM prediction can be accommodated. However, the upper limit on the  $\text{BR}(b \rightarrow s\gamma)$  imposes a severe constraint on the SUSY parameter space since the supersymmetric contribution adds to the SM prediction.

For  $\mu < 0$  and  $M_X > M_{\text{GUT}}$ , we find that the WMAP areas allowed due to  $\chi - \tilde{\tau}$  coannihilations at low values of  $\tan \beta$  are excluded by the  $b \rightarrow s\gamma$  higher bound. However, at larger values of  $\tan \beta$ , the areas with resonant annihilations are marginally allowed, as we can see in figure 8.

As already underlined, these results are very sensitive to  $\lambda_N$ . Once we decrease it by lowering  $M_N$  (so that, from the see-saw condition,  $m_{\nu_3}$  remains 0.05 eV) we allow larger regions of the parameter space for negative  $\mu$  as well. In both cases, however, the values of the parameter  $\delta$  have to be very small ( $\delta < 0.05$ ); therefore, negative  $\mu$  is not compatible with large charged lepton Yukawa mixing.

## 6 Conclusions

We revisited the WMAP dark matter constraints on  $b - \tau$  Yukawa Unification in the presence of massive neutrinos for both the CMSSM and an mSUGRA-like scenario with universal soft terms at  $M_X > M_{\text{GUT}}$ . Large lepton mixing, as indicated by the data, modifies the predictions for the bottom quark mass, and enables Yukawa unification also for



**Figure 8.**  $(m_0, M_{1/2})$  plane for  $\tan\beta = 40$ ,  $A_0 = 0$ , and  $\mu < 0$ . On the left panel we take the right-handed neutrino scale  $M_N = 6 \times 10^{14}$  GeV, while on the right panel  $M_N = 10^{13}$  GeV. Here,  $m_b(M_Z)$  and  $\alpha_s(M_Z)$  are set to their central values. The lines follow the same notation as figure 4.

large  $\tan\beta$  and for positive values of  $\mu$ , which were previously disfavoured. The larger the Dirac neutrino Yukawa couplings, the larger the effects. The correlations implied by  $b - \tau$  unification significantly constrain the various possibilities for all fermion (and consequently sfermion) masses and mixings, and other observables. In our work, the allowed parameter space is derived by checking the consistency with all known observables, including the most recent ones for neutrino masses. If one drops these (theoretically motivated) correlations, it is clear that it becomes easier to obtain solutions, which are however less predictive.

A direct outcome is that the allowed parameter space for neutralino dark matter also increases, particularly when the effects of large lepton mixing are combined with runs above  $M_{\text{GUT}}$ . Summarising, we find the following:

- $b - \tau$  unification is only allowed for  $\mu > 0$  in the presence of large charged lepton mixing, which is motivated by the experimental data of the recent years.
- For  $\mu < 0$  we still find values compatible with an exact  $b - \tau$  unification at  $M_X$ . However, for large Dirac neutrino Yukawa couplings and mixing arising dominantly from the charged lepton sector in the basis where the down-quark mass matrix is diagonal, the space of parameters compatible with WMAP is i) only marginally compatible with the upper bound on  $b \rightarrow s\gamma$  and ii) entirely excluded on the grounds of the  $(g-2)_\mu$  observations, if we use  $e^-e^+$  data to estimate the SM vacuum polarization contribution.

Interestingly enough, it turns out that the cosmologically favoured parameter space also implies lepton flavour violating (LFV) rates that are very close to the current experimental bounds since in addition to the flavor mixing introduced by the see-saw, we have the double triplet mass splitting induced at GUT from the SU(5) runs above this scale [44].



## Acknowledgments

The authors would like to thank the European Network of Theoretical Astroparticle Physics ENTApP ILIAS/N6 under contract number RII3-CT-2004-506222 for financial support. The research of S. Lola and P. Naranjo is funded by the FP6 Marie Curie Excellence Grant MEXT-CT-2004-014297. The work of M.E.G and J.R.Q is supported by the Spanish MEC project FPA2006-13825 and the project P07FQM02962 funded by Junta de Andalucia.

## A RGE for Yukawa couplings

In this appendix we summarize the RGE's that are most relevant for the purposes of the work addressed in this paper. For runs above the GUT scale the equations involving the Yukawa couplings and the soft mass terms corresponding to the  $\mathbf{10}$  and  $\bar{\mathbf{5}}$  representations of SU(5), for the  $3^{rd}$  generation, take the form [23]

$$16\pi^2 \frac{d\lambda_N}{dt} = \left[ -\frac{48}{5}g_5^2 + 7\lambda_N^2 + 3\lambda_t^2 + 4\lambda_b^2 \right] \lambda_N, \quad (\text{A.1})$$

$$16\pi^2 \frac{d\lambda_d}{dt} = \left[ -\frac{84}{5}g_5^2 + 10\lambda_d^2 + 3\lambda_t^2 + \lambda_N^2 \right] \lambda_d, \quad (\text{A.2})$$

$$16\pi^2 \frac{d\lambda_t}{dt} = \left[ -\frac{96}{5}g_5^2 + 9\lambda_t^2 + 4\lambda_d^2 + \lambda_N^2 \right] \lambda_t, \quad (\text{A.3})$$

$$16\pi^2 \frac{dm_{\mathbf{10}}^2}{dt} = -\frac{144}{5}g_5^2 M_5^2 + (12\lambda_t^2 + 4\lambda_d^2) m_{\mathbf{10}}^2 + 4 \left[ (m_{\bar{\mathbf{5}}}^2 + m_h^2) \lambda_d^2 + A_d^2 \right] + 6 \left( \lambda_t^2 m_h^2 + A_t^2 \right), \quad (\text{A.4})$$

$$16\pi^2 \frac{dm_{\bar{\mathbf{5}}}^2}{dt} = -\frac{96}{5}g_5^2 M_5^2 + 2 \left( 4\lambda_d^2 + \lambda_N^2 \right) m_{\bar{\mathbf{5}}}^2 + 8 \left[ (m_{\mathbf{10}}^2 + m_h^2) \lambda_d^2 + A_d^2 \right] + 2 \left( \lambda_N^2 m_h^2 + \lambda_N^2 m_{\mathbf{1}}^2 + A_N^2 \right) \quad (\text{A.5})$$

For runs from  $M_{\text{GUT}}$  to  $M_N$ , the equations for the Yukawa matrices are:

$$16\pi^2 \frac{d\lambda_N}{dt} = - \left[ \left( \frac{3}{5}g_1^2 + 3g_2^2 \right) I_3 - (4\lambda_N^2 + 3\lambda_t^2 + \lambda_\tau^2) \right] \lambda_N, \quad (\text{A.6})$$

$$16\pi^2 \frac{d\lambda_\tau}{dt} = - \left[ \left( \frac{9}{5}g_1^2 + 3g_2^2 \right) I_3 - (4\lambda_\tau^2 + 3\lambda_b^2 \lambda_N^2) \right] \lambda_\tau, \quad (\text{A.7})$$

$$16\pi^2 \frac{d\lambda_t}{dt} = - \left[ \left( \frac{13}{5}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2 \right) I_3 - (6\lambda_t^2 + \lambda_b^2) + \lambda_N^2 \right] \lambda_t \quad (\text{A.8})$$

Since the neutrino has no coupling to the bottom quark, the Yukawa matrix corresponding to the latter remains unchanged with respect to the MSSM case.

In section 2 semi-analytical expressions for the small  $\tan\beta$  regime were given. In that case, only the top and the Dirac-type neutrino Yukawa couplings can be large at the GUT scale.

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